Exhibit 4

EXHIBIT N

Exhibit B (Supplemental)

U.S. Pat. No. 8,407,273 Claim 53

53. A device:

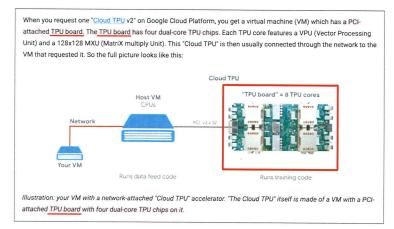
comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,

wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

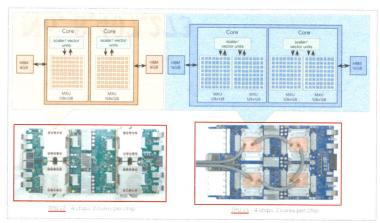
wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

SUPPLEMENTAL INFRINGEMENT EVIDENCE

As demonstrated below, the Accused Products include multiple components that, separately and independently, meet all the requirements of the claimed "device.":



https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#2



https://cloud.google.com/tpu/docs/system-architecture

SUPPLEMENTAL INFRINGEMENT EVIDENCE

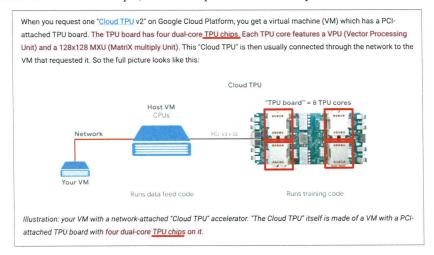
53. A device:

comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value.

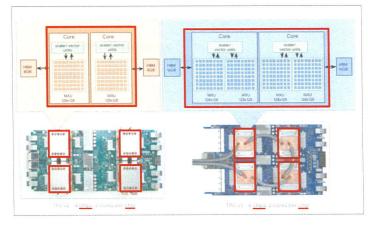
wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

As demonstrated below, the Accused Products include multiple components that, separately and independently, meet all the requirements of the claimed "device." For example, a "TPU Chip" satisfies these requirements:



https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#2



https://cloud.google.com/tpu/docs/system-architecture

See also generally Norrie et al., "Google's Training Chips Revealed: TPUv2 and TPUv3" (Presented at HotChips Conference, Aug. 2020)

53. A device:

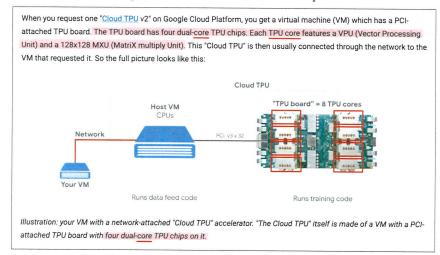
comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value.

wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

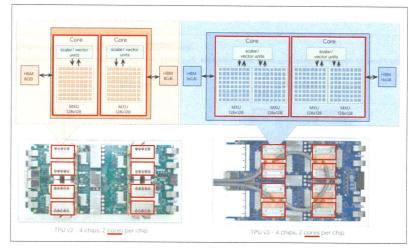
wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

SUPPLEMENTAL INFRINGEMENT EVIDENCE

As demonstrated below, the Accused Products include multiple components that, separately and independently, meet all the requirements of the claimed "device." For example, a "TPU Core" satisfies these requirements:



https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#2



https://cloud.google.com/tpu/docs/system-architecture

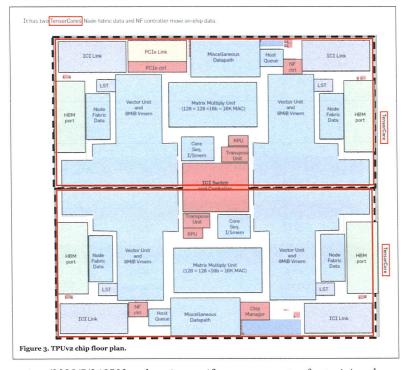
53. A device:

comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,

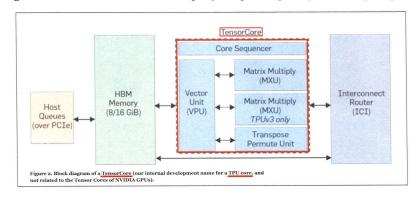
wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

SUPPLEMENTAL INFRINGEMENT EVIDENCE



https://cacm.acm.org/magazines/2020/7/245702-a-domain-specific-supercomputer-for-training-deep-neural-networks



Id.

See also GOOG-SING-SC-000001-454.

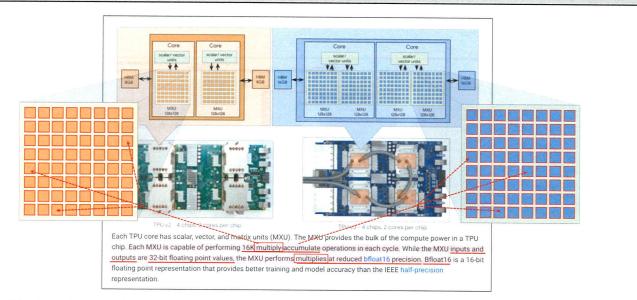
53. A device:

comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,

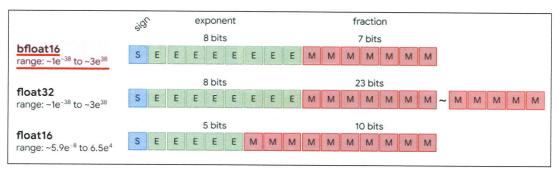
wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

SUPPLEMENTAL INFRINGEMENT EVIDENCE



https://cloud.google.com/tpu/docs/system-architecture



https://cloud.google.com/tpu/docs/bfloat16

SUPPLEMENTAL INFRINGEMENT EVIDENCE

53. A device:

comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,

wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1.000.000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

Systolic array

The MXU implements matrix multiplications in hardware using a so-called "systolic array" architecture in which data elements flow through an array of hardware computation units. (In medicine, "systolic" refers to heart contractions and blood flow, here to the flow of data.)

The basic element of a matrix multiplication is a dot product between a line from one matrix and a column from the other matrix (see illustration at the top of this section). For a matrix multiplication Y=X*W, one element of the result would be:



Illustration: the MXU systolic array. The compute elements are multiply-accumulators. The values of one matrix are loaded into the array (red dots). Values of the other matrix flow through the array (grey dots). Vertical lines propagate the values up. Horizontal lines propagate partial sums. It is left as an exercise to the user to verify that as the data flows through the array, you get the result of the matrix multiplication coming out of the right side.

https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#2

See also GOOG-SING-SC-000001-10, 13-30, 33-61, 228-292, 315-373, 396-444, 449-454.

SUPPLEMENTAL INFRINGEMENT EVIDENCE

53. A device:

comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value.

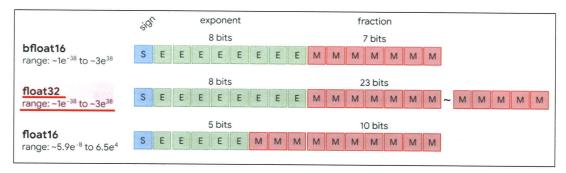
wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

https://cloud.google.com/tpu/docs/system-architecture

• "Each TPU core has scalar, vector, and matrix units (MXU). The MXU provides the bulk of the compute power in a TPU chip. Each MXU is capable of performing 16K multiply-accumulate operations in each cycle. While the MXU inputs and outputs are 32-bit floating point values, the MXU performs multiplies at reduced bfloat16 precision. Bfloat16 is a 16-bit floating point representation that provides better training and model accuracy than the IEEE half-precision representation."

- "The following figure shows three floating-point[] formats
 - fp32 IEEE single-precision floating-point
 - fp16 IEEE half-precision floating point
 - bfloat16 16-bit brain floating point"

https://cloud.google.com/tpu/docs/bfloat16



Id.

See also GOOG-SING-SC-45-61, 435-444, 449-454.

wherein the number of LPHDR execution

SUPPLEMENTAL INFRINGEMENT EVIDENCE

53. A device:

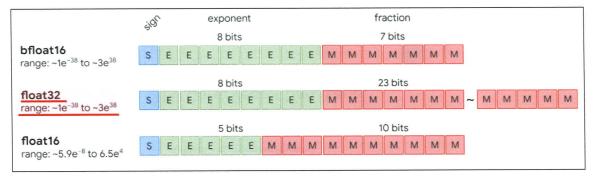
- comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

"Each TPU core has scalar, vector, and matrix units (MXU). The MXU provides the bulk of the compute power in a TPU chip.
Each MXU is capable of performing 16K multiply-accumulate operations in each cycle. While the MXU inputs and outputs are 32-bit floating point values, the MXU performs multiplies at reduced bfloat16 precision. Bfloat16 is a 16-bit floating point representation that provides better training and model accuracy than the IEEE half-precision representation."

https://cloud.google.com/tpu/docs/system-architecture

- "The following figure shows three floating-point[] formats
 - fp32 IEEE single-precision floating-point
 - fp16 IEEE half-precision floating point
 - bfloat16 16-bit brain floating point"

https://cloud.google.com/tpu/docs/bfloat16



Id.

• "Because general-purpose processors such as CPUs and GPUs must provide good performance across a wide range of applications, they have evolved myriad sophisticated, performance-oriented mechanisms. As a side effect, the behavior of those processors can be difficult to predict, which makes it hard to guarantee a certain latency limit on neural network inference. In contrast, TPU design is strictly minimal and deterministic as it has to run only one task at a time: neural network prediction. You can see its simplicity in the floor plan of the TPU die."

https://cloud.google.com/blog/products/gcp/an-in-depth-look-at-googles-first-tensor-processing-unit-tpu (emphasis in orig.)

• "In mathematics, computer science and physics, a <u>deterministic</u> system is a system in which <u>no randomness</u> is involved in the development of future states of the system. A <u>deterministic</u> model will thus <u>always produce the same output</u> from a given starting condition or initial state."

https://en.wikipedia.org/wiki/Deterministic_system

- For each of the possible valid inputs to the multiplication operation performed by the multipliers within the MXU, Singular has computed the result and compared it to the result of an exact mathematical calculation performed on the same inputs. The results of this test showed that for more than 10% of the possible valid inputs, the numerical value represented by the output signal of each MXU multiplier differs by more than 0.2% from the result of an exact mathematical calculation performed on the same inputs.
- See also GOOG-SING-SC-000001-10, 13-30, 33-61, 228-292, 315-373, 396-444, 449-454.

SUPPLEMENTAL INFRINGEMENT EVIDENCE

53. A device:

comprising at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value.

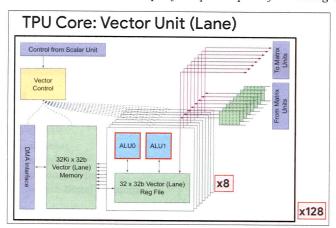
wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

The Accused Products independently meet this claim limitation for each "device" identified above:

TPUv3 is 6% larger in that same technology. TDP stands for Thermal Design Power. The Volta has 80 symmetric multiprocessors.				
Feature	TPUv1	TPUv2	TPUv3	Volta
Peak TeraFLOPS/ Chip	92 (8b int)	46 (16b) 3 (32b)	123 (16b) 4 (32b)	125 (16b) 16 (32b)
Network links x Gbits/s/Chip	-	4 x 496	4 x 656	6 x 200
Max chips/supercomputer	-	256	1024	Varies
Peak PetaFLOPS/supercomputer		11.8	126	Varies
Bisection Terabits/supercomputer		15.9	42.0	Varies
Clock Rate (MHz)	700	700	940	1530
TDP (Watts)/Chip	75	280	450	450
TDP (Kwatts)/supercomputer	961 ₍ -13)	124	594	Varies
Die Size (mm²)	<331	<611	<648	815
Chip Technology	28nm	>12nm	>12nm	12nm
Memory size (on-/off-chip)	28MiB/8GiB	32MiB/16GiB	32MiB/32GiB	36MiB/32GiB
Memory GB/s/Chip	34	700	900	900
MXUs/Core, MXU Size	1 256x256	1 128x128	2 128x128	8 4x4
Cores/Chip	1	2	2	80
Chips/CPU Host	4	4	8	8 or 16

https://cacm.acm.org/magazines/2020/7/245702-a-domain-specific-supercomputer-for-training-deep-neural-networks/full text.



Norrie et al., "Google's Training Chips Revealed: TPUv2 and TPUv3" (Presented at HotChips Conference, Aug. 2020)

See also GOOG-SING-SC-000001-454.